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PRESENTISM AND THE SPANS OF TIME*

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Presentists, who believe that only present entities exist, face a problem of how to analyse tensed plural quantification (Lewis 2004, Brogaard 2007). The idea, in broad outline, is that presentists can't employ the usual method for analysing tensed singular quantification, using primitive 'slice' tense operators, to analyse plurals. One option is to introduce a new theoretical primitive: a 'span'-operator. But there are reasons to worry about this option. For one, we might agree with Lewis (2004) that span-operators are ill-behaved or introduce unpalatable complexity. For another, we might wish to avoid any additional theoretical primitives, regardless of whether spans can earn their keep alongside slices.

In this paper, we offer a strategy for presentists to address the problem that requires only slice-operators. Presentists who wish to avoid span-operators, for whatever reason, should find our proposal attractive. The plan for the paper is simple. First, we outline the problem of tensed plural quantification in more detail (§1). Then, we show how presentists can address the problem with slice-operators (§2). Finally, we deal with some anticipated objections (§3).

1. *Tensed Quantification*

The Library of Alexandria doesn't exist now. So, it doesn't exist at all, given presentism—the view that only present objects exist. Nonetheless, many will agree (deferring to historians) that:

(1) There was a library at Alexandria.

The standard presentist analysis of this claim involves the unrestricted quantifier (i.e. 'there is') within the scope of a past-tense operator (i.e. 'It was the case that' or, simply, *WAS*). Thus:

(2) *WAS* (there is a library at Alexandria).

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Presentists can endorse (2), without thereby accepting wholly past entities, since quantifiers within the scope of tense operators aren't ontologically committing (Sider 2001: 15).

The past-tense operator '*WAS*' in (2) is a primitive slice-operator; a tense operator that picks out one instant, one 'slice' of the past (Sider 2001: 26). Hence, (2) has an informal gloss: it was the case, at some instant, that there is a library at Alexandria. The slice-operator approach is fit for purpose when dealing with singular quantification, such as (1), but it faces problems when we consider plural quantification—quantification over a plurality of past objects at a plurality of past times.

Lewis (2004) develops this problem for presentism. We borrow one of his examples:

(3) There have been two kings (of England) named Charles. (cf. 2004: 5)

Here is the standard analysis, using a slice-operator:

(4) *WAS* (there are two kings named Charles).

The analysis fails. For one, (4) isn't a good way to understand (3); when we assert (3) we don't mean that it was the case, at an instant, that there are two kings named Charles. For another, (4) is false; at no time in history have there been two English kings named Charles (2004: 5). The slice-operator restricts us to one time; plurals appear to require many times or a span of times.

Two avenues of reply have been suggested. One option is to persevere with the slice-operator approach, to try to make sense of talk of spans by using talk of individual times. Another option is to introduce a new operator, i.e. a primitive span tense operator, in addition to the slice-operator (e.g. Bourne 2007, Brogaard 2007). We favour the former option. We are slicers.

Why reject span operators? We hesitate to endorse a further theoretical primitive when we think that the work can be done adequately with only slice-operators. All things being equal, we should reduce the number of primitives, whenever possible.[1] If presentists can do without either primitive spans or slices, they should do so. And, as it happens, presentists can't do without slice-operators. Let us explain.

Contrast the past-tense slice-operator, 'It was the case that' (*WAS*), with a past-tense span-operator, 'It has been the case that' (*HAS*). The span-operator doesn't pick out one instant—one slice—but many instants.

[1] One option that doesn't require a primitive span-operator is to account for spans in terms of quantification over *ersatz* times (Bourne 2007). But this is only open to ersatz presentists, such as Bourne (2006, 2007). Our project here is to sketch a solution that's open to all presentists.

There are problematic sentences that involve span-operators and the sensible way to resolve such problems involve slice-operators. (Hence, presentists can't do without slice-operators.) Again, we borrow an example from Lewis:

(5) *HAS* (it is raining and the sun is shining). (cf. 2004: 12)

This is ambiguous. It might mean that there has been a temporal interval throughout which it has both been the case that it rained and the sun shone. Or, it might mean that there was an interval, throughout a sub-interval of which the sun shines, and throughout another it rains (2004: 12–13). In any case, if the semantics of the operator are ambiguous, it doesn't give us a sensible semantics for span-talk.

Rather than endorse one of either span-operators or slice-operators, Brogaard (2007) thinks that we should accept both. This allows us to remove ambiguity in sentences involving span-operators. For instance, our ambiguous sentence (5) can be clarified as:

(6) *HAS* (it is raining),
& *HAS* (the sun is shining),
& ~ *WAS* (it is raining and the sun is shining). (cf. 2007: 74)

The combination of operators is powerful and yields greater precision. On this point, we agree broadly with Brogaard. But, we think that presentism can do without the additional operator. We are slicers and our manifesto is this: everything that the presentist needs to do can be done with slice operators and without the aid of span operators.

2. Presentism with Slices

The original difficulty with analysing span-talk via a slice-operator is that the slice-operator restricts us to one time whereas tensed plural quantification requires a span of times. For presentists to persevere with only slice-operators, we must make sense of span-talk by using talk of individual times. Somewhat loosely, we can describe this approach as analysing span-talk in terms of the various slices that would make up a span, were such a thing to exist.

Hence, a putative analysis for our tensed plural quantification, (3), is as follows:

(7) *WAS* (there is a king named Charles & *WAS* (there is another king named Charles)).

This is a 'nested translation' of (3); the second slice-operator is nested within the scope of the first. Presentists must analyse (3) in nested fashion; they can't make sense of 'another' unless it occurs within the scope of the first operator (Lewis 2004: 5–6; Brogaard 2007: 72–73). Between them, these two operators generate a small 'span' of time out of the two slices described. And, so far as it goes, this is satisfactory: unlike (4), (7) is (plausibly) what we mean when we assert (3) and (7) is true. Even so, there are problems with applying this analysis to all cases of tensed plural quantification. Our example involving English kings is all too convenient: there have never been multiple kings at one time; and, there have been only finitely many. But this isn't the case for all entities, and presentists must deal with many troublesome sentences, some of which will take the form, 'There have been *n F*s', in which the value of *n* is infinite, the value of *n* is unspecified or underspecified (e.g., 'There have been *some F*s), and/or some of the *F*s exist simultaneously.

Consider, for instance:

(8) There have been two books on the desk.

This is a simple example which introduces a surprising amount of complexity, since the issue of whether the books are on the desk instantaneously or simultaneously is unsettled. An extra disjunct is required to make sense of the claim. Thus:

(9) *WAS* (there is a book on the desk, & either: there is another book on the desk, or: *WAS* (there is another book on the desk)). (cf. Lewis 2004: 6)

Here the slice-operator analysis is satisfactory, but unlovely. And things are about to get more unlovely.

Matters appear much worse in the case of span-talk with an infinite number of entities. For instance:

(10) There have been uncountably many electrons.

As Lewis puts it, the trouble is that the translation of (10) 'requires a construction with tense operators nested *ad infinitum*' (2004: 7). Of course, we have a sense of how the analysis would go—it would start like (9), and increase in complexity (to put it mildly)—but we couldn't complete it.

Lewis (2004: 7) argues that the 'unsuspected complexities' that arise for what we call a 'slicer' approach, gives presentists a reason to reject it—though does note that this doesn't constitute a 'decisive refutation'; rather, it is a cost to be born. We think that the cost is quite light and the payoff sufficiently significant as to out-weigh it. We advocate slicing.

Before we make our positive case, let's get clearer on slice-operators. Slicer-presentists must be able to understand all talk about past times using slice-operators. This brings a problem to light. Dealing with it allows us to get clearer on the structure of the operator. Consider two claims:

(11) *WAS* (there are dinosaurs).

(12) ~ *WAS* (there are dinosaurs).

Both are clearly true, given a particular time of evaluation: (11) is true of 65 million years ago, (12) is true of 5 minutes ago, for instance. Since *WAS* is a slice-operator, we must be clear upon the way in which the claims are differentiated. Suppose that we regiment talk about the time immediately prior to the time immediately prior to the present, using the operator, we would write something of the form: *WAS* (*WAS* (…)). As a shorthand, we can represent this using a single term, $WAS_{\#}$ (…), where # takes the value of the number of iterations of the operator. To clarify: WAS_1 (…) refers to the time that was present immediately before the actual present. WAS_2 (…) refers to the moment that occurred most immediately before the time picked out by WAS_1 (…), and so on. Of course, in most cases where 'was' is deployed, the correct interpretation of 'was' is generated either by context, or by some explicit content of the sentence in which 'was' is embedded. Thus, when one asserts that there were dinosaurs, what is said is something of the form:

(13) $WAS_{\text{65 million years ago}}$ (there are dinosaurs).

To be formally accurate, we would replace the '65 million years' in # with the number of instants. On the (defeasible) assumption that the Planck time (10^{-43} seconds) is the smallest physically possible temporal unit, the number of instants inserted in lieu of '65 million years' would be number of seconds in a year (31,536,000) multiplied by the number of instants in a second (1,043). We leave it as '65 million years' in (13) since it is easier. So long as we can put moments of time into 1:1 correspondence with the real numbers, and use the latter to name the former, the construction that we offer is viable.[2]

With that in hand, we now turn our attention to the details of a good slice.

2.1 Slice and Paraphrase

Let's consider a case (alluded to above) in which what we say in natural language, although true and meaningful, seems rather imprecise. That is:

(14) There have been some kings called Charles.

[2] We are grateful to a reviewer for pressing us to say more here and for suggesting a point of clarification. We say more about the case of continuous time in §3.2 as it threatens a further complication.

Since (14) isn't specific as to when the kings lived, we don't know (in principle) how to construct the required translation. Thus, we cannot just *slice*—we can't just use slice operators. The concern isn't that we lack the computational power to translate, the concern is that we can't use slice-operators to capture what is said. We don't know, even in principle, what the translation will look like. How to solve this problem?

Consider a crude paraphrase of (14). The paraphrase isn't right, but by examining it some important issues can be brought out. Suppose that we render the claim that there were some kings in first-order logic, with 'K' as the predicate 'is a king'. Thus:

(15) $WAS_{\#}$ ($\exists x\ \exists y$ ((Kx & Ky) & $x \neq y$))

This is our starting point. There are clearly problems with it. One might object that it doesn't capture what's intended by (14); (14) is intended to say that the kings existed at different times. Because (15) doesn't say this, (15) can't be a good paraphrase of (14). This is not a promising start.

But slicers are new on the scene. We advise that stealing the resources of others may be the best way forward and that this theft will aid is in our travails.

A brief detour will explain what we wish to repurpose to our own ends. Let's move to another topic—a problem for mathematical nominalism. The motivation for our diversion should be obvious. The mathematical nominalist doesn't believe in the existence of numbers (as abstracta). Consequently, the nominalist must develop strategies for dealing with cases involving 'hard-to-paraphrase' mathematical claims. What we have in the presentist case are hard-to-paraphrase temporal claims. Exploring nominalist resources is a sensible strategy for the slicer.

In what follows, we borrow a case from Melia (1995) and look to reapply his strategy for dealing with the mathematical case, to the temporal case.

Melia discusses two problem sentences:

(16) The average mum has 2.4 children.
(17) The number of Argle's fingers equals the number of Bargle's toes.

Nominalists want to deny the existence of both average mothers (and their 2.4 children) as well as the number of Argle and Bargle's digits. But, then, the two claims are concerning: (16) apparently entails that the average mother exists, (17) apparently implies that a number exists.

Melia suggests a nominalist paraphrase. In the case of (16), once we've counted all the mothers and their offspring, we should write down a sentence in first-order logic that reports (e.g.) how many *F*s and *G*s there are (how many mothers and offspring there are), which existentially commits us to nothing more than *F*s and *G*s (1995: 225)—mothers and offspring. The strategy is serviceable and what such a paraphrase looks like is familiar enough.

But just as in the temporal case, there are troubles lurking. Melia explains:

> 'Consider … the sentence 'the average star has 2.4 planets'. Perhaps the implications for the concrete world that this sentence has are indeed correct – suppose that there are precisely twentyfour zillion orbiting planets and ten zillion stars. But whilst we may have very good evidence that the average star has 2.4 planets, we may not have any evidence at all that there are precisely twentyfour zillion planets and ten zillion stars. And our chances of counting up all the stars and planets are, to say the least, slim. Indeed, I think that our ignorance in this case is ineliminable, and so the manoeuvre considered above is not open to us. When our ignorance is ineliminable, the programme simply cannot be carried through.' (1995: 226)

Thus we find that our best theory ought to include the claim that 'the average star has 2.4 planets' and we have no way to paraphrase it. If we have no paraphrase, we have no way of eliminating the unwanted ontological commitment to 'average stars'. In addition, suppose that we thought that:

(18) There are uncountably many electrons.

How do we paraphrase it? The problem is clear. We can't write down the required paraphrase. This is analogous to the problem facing presentists. Just like the nominalist, presentists wishing to deal with spans are faced with a situation where they must provide a paraphrase for a case involving uncountably many entities, as in (10).

We endorse Melia's (1995: 228) conclusion that, merely because our theory appears to quantify over some property or entity, doesn't entail that such a thing exists; merely that we cannot state a complete paraphrase of some talk doesn't mean that the world is other than would be described by such a paraphrase, were it to be stated. In the case that Melia discusses we should simply say that there's a paraphrase, and that the paraphrase is being described by the locution 'the average star has 2.4 planets'.

In our view, if it turns out that we cannot write-out the paraphrase in logical form, that doesn't mean that what would be paraphrased has anything over and above a 'paraphrasable' content. The paraphrase is just a shorthand way of expressing *that*. This is *why* we have developed natural language in the way that we have;

it gives us an excellent way of expressing first-order logical commitments that, otherwise, would be extremely hard (perhaps even impossible) to express.

We can explore this in the mathematical case just a little. Many of us are competent users of 'number-talk' and we had to learn how number-talk worked. Let's suppose that we learn in the following way:

(19) To say 'there is one F' is to say:
$\exists x\ (Fx\ \&\ \exists y\ (Fy \rightarrow y=x))$

(20) To say 'there are two Fs' is to say:
$\exists x\ \exists y\ ((Fx\ \&\ Fy)\ \&\ x \neq y)\ \&\ \exists z\ (Fz \rightarrow z = x\ \ v\ \ z = y)$ *Etc.*

This isn't how we learn to deploy numbers. To suggest otherwise is odd. We don't learn the logical calculus when we first learn to manipulate arithmetic symbols. Many (all?) learn to use numbers by learning to count entities that fall under a particular sortal. Thus, we learn to count that there are (e.g.) "one, two, three; three pens on the desk". In counting this way, we recognise that there was a pen, another pen, distinct from the former; and another, distinct from both of the preceding. This, or so we claim, is what's going on when we learn to count—we do so by counting particulars (see Schaeffer et al. 1974). But what we've just described is precisely what's captured by the predicate calculus. For instance, (20) says precisely that there is an *F*, and there is another *F*, and that these are the only *F*s.

Suppose that this means of counting and understanding number-talk is available to us within the lower reaches of the natural numbers. Matters get more difficult quickly, but only because of computational power. Writing out the paraphrase for 'there are 87 cats' is a laborious task, certainly, but conceptually it's no more challenging than writing out the paraphrase for 'there are 3 cats'. So, what happens is this: we use number-talk to articulate our claims. This has obvious advantages. Rather than saying 'there is a lion, and another distinct lion, and another distinct from both of the others, and these are all the lions, and the lions are running toward us', we say 'THERE ARE THREE LIONS RUNNING TOWARDS US!'. We learn this lesson well. Number-talk can be used as a shorthand for expressing claims about distinct existents. As the lion case demonstrates, that's potentially an extremely useful capacity to have available.

There's another sense in which this capacity is useful to us. Not only is this shorthand advantageous when being pursued by hungry lions, it's also useful when we want to make claims like (10) or (18). If there are (or have been) uncountably many electrons, we cannot write down the required paraphrase. So, what should we say? Our number-talk is as we described it above; to say that there are two cats is to say little more than what's said in (20). But, as we've seen, number-talk is more straightforward to use than sentences like (20). It offers us a way of capturing a particular idea and expressing it quickly and straightforwardly. And it's for similar reasons that we use terms such as 'uncountably many'. They allow us to describe the *F*s; the electrons in this case.

But merely that we say things using a natural language, and find it more expressive than a first order predicate calculus, and more *useful* than the logical calculus, does not mean that we get to read our ontological commitments from our natural language. If (as we speculate) we learned to count relative to a sortal, and in the way we described, it would be natural to view natural language as means of encoding complex claims *of that sort*. That is, natural language is often used to make very complex claims about there being an F, and another, F, such that the Fs are not identical, and another F,…and so on. The great virtue of natural language is that it makes all of this *so easy*. But, once again: merely that we cannot state a complete paraphrase of some talk doesn't mean that the world is other than would be described by such a paraphrase, were it to be stated.

The plausibility of the general idea can be brought out by consideration of other cases. Suppose that space is point-like and continuous; there are infinitely many space-time points between any two points, *a* and *b*. To describe the motion of an object O from *a* to *b* we would naturally say:

(21) O travelled from *a* to *b*.

But, given the metaphysical structure of the world described, we would have to concede that, in fact, the motion glibly described in (21) is just a simplified way of saying that O moved as follows (with all spatial points between *a* and *b* named, $s_1 \ldots s_n$):

(22) O moved from s_1 to s_2, from s_2 to s_3 … and from s_n to *b*.

Of course, the paraphrase gestured towards in (22) can't be completed. But it seems unreasonable to conclude that this shows that O can't move or space can't be point-like and continuous. The lesson from (22) is that we use sentences like (21) to say something that we can't in the paraphrase, because the paraphrase would be too long. Nonetheless, that infinitely long description is captured in some sense by (21). It's one of the great advantages of natural language that it's capable of capturing such infinitely long strings in a finite structure.

And, to return to the start of this section, that we know how, in principle, to state a paraphrase of 'the average star as 2.4 planets' (count up the stars, count up the planets, specify that for every 24 planets there are 10 stars) means that we should conclude that language is a means of expressing an extremely long, complex (and deadly boring) sentence of first-order logic. Its expressive power is one of the reasons we use natural language to communicate, rather than something that can be as unwieldy as predicate logic.

2.2 Back to Tensed Quantification

All of the above helps us with tensed plural quantification and the problem of span-talk. Recall, what we find is that there are cases in language where we appear to quantify over spans even though no such things exist. The moral extracted from the nominalist cases is that paraphrase strategies ought to be regarded as successful if we know, in principle, how to construct a paraphrase of such talk. One original problem case for presentists is how to make sense of:

(10) There have been uncountably many electrons.

We now have a better sense of our answer. (10) requires an infinite nesting of operators and quantifiers, using only slice-operators. We cannot write down such a translation. But this is unsurprising. Indeed, it's what we should expect. A sentence in which we talk of uncountably many entities will be one for which we cannot provide a translation. This is just what we found in discussion of Melia's stars and planets. This poses no problem, of course, provided we know, in principle, how to construct it. And we do. Allow that 'E' stands for 'is an electron', then: WAS ∃x (Ex & WAS ∃y (Ey & ~(x=y)…. But just keep going.

Thus, the idea is that we have slice-operators and presentists can use them to paraphrase span-talk in most cases. There are some cases where we can't *write* the paraphrase; the paraphrase required is too long. But, rather than take this as a problem, we simply note that we know in principle how to construct the paraphrase. We understand what's said by the natural language claim and what's being said commits us to no more than slices and electrons at those slices. We use natural language to make these claims because it's so useful. So, we say (10) because we lack the resources to say (write) the paraphrase and it's useful to be able to say such things. This is our slicer position.

3. Objections and Replies

Now we move to considering objections to our slicer-presentism. We do so in the hopes of both offering a response as well as giving the reader a better feel for our view. To begin, recall the earlier case when it's indeterminate how many entities are quantified over. That is:

(14) There have been some kings called Charles.

One might object that it remains unclear how to paraphrase (14) and that our strategy doesn't give us the tools with which to paraphrase, even in principle. To paraphrase (14), we draw upon resources introduced in §2. Thus:

(23) $\mathit{WAS}_{\#n}$ (∃x: x is a king called Charles), &

$\mathit{WAS}_{\#m}$ (∃y: y is a king called Charles), &

~ $\mathit{WAS}_{\#p}$ (∃x ∃y: x is a king called Charles & y is a king called Charles & x ≠ y).

Does this approach—translating with only slice-operators—generate such complexity that presentists should abandon the slice-only view, as Lewis (2004) suggests?

We think not. The superficially simple (14) is *in fact* misleading in form; it's used to make a complex and highly ambiguous claim. What a paraphrase does is make the expression fully precise. When making precise a complex and highly ambiguous expression, we should *expect* the paraphrase to be complex! How else to capture precisely all of the dimensions of ambiguity?[3] We shouldn't think it at all surprising that the correct paraphrase is long and complex. In fact, since sentences like (23) can be used to express claims that are themselves already incredibly complex, even from only a cursory inspection, i.e. claims about a vast number of times, a potentially limitless number of distinct entities, and the location of said entities at said times. It's surprising to us that Lewis thought this a surprising result, nor do we see any reason to think it 'bad news'. A complex paraphrase for a complex expression sounds right to us.

But perhaps you think that this still spells trouble because the complexity of the paraphrase still serves to rebuff common-sense. It's surely no part of common-sense that a simple-seeming sentence deserves such a complex translation.

Once more, we disagree. Begin by noting that the view of the 'common man' (as Lewis puts it) is most likely that there's no need for such translations. Translations are, in this context, philosophical tools and the view of the common person is presumably that, unless presented with a special reason, we have no need to translate sentences, the meaning of which seems perfectly clear (when we take notice of context, intention, and so on).

What else might be said? It's reasonable to treat the forgoing exercise as one in which we engage in the practice of precisifying claims of English. Consider (14) once more. Suppose that someone just uttered it and that you want more information about what was meant. One might well ask: 'can you be more specific by what you mean by (14)'?

[3] To give a homespun case: 'I promise to do everything that I can to help you' is superficially simple but, as a mere moment's reflection reveals, *incredibly* complex to disambiguate.

The speaker may answer 'no'. If that's true, then it's surely because each of the options that would be stated in the long disjunction is left open. If we mean to leave open each combination, it's no problem that they are being explicitly mentioned by the specification of the meaning of the sentence. In that case, the complexity is apt.

But what if the answer is 'yes'? Well, then we suspect that one of a number of things might be pressed. Imagine that we want to press our interlocutor. What will they say to make matters more precise? We suspect that they will say that, for instance, rather than simply being several, there were in fact two kings of England named Charles (and, moreover, that one reigned between 1625 and 1649, and that the other reigned between 1660 and 1685). With all that information in play, it's no problem at all to specify, using a slice analysis, what needs to be said, even though it will, once again, require an extremely long paraphrase. But since the information to be encoded is rather complex, and since paraphrases are in general rather cumbersome (as we saw above and thus explaining why we don't use them in everyday life) this is of no surprise either.

3.1 'Usually'

Let's turn now to a final putative problem case from Brogaard (2007), raised to demonstrate one of the appealing features of span-operators. Brogaard claims that taking span-operators as primitive makes it easier to translate sentences involving cases of adverbs of quantification. The example given is this:

(24) When I was a child I usually behaved well.

Brogaard (2007) explains the difficulty as follows:

> 'Eternalists take adverbs of quantification – such as 'usually', 'generally', 'always' – to quantify over cases, where cases are *n*-tuples of individuals, times or events (Lewis 1975). The cases quantified over in [(24)] are times at which I am a child. But if presentism is true, these times do not exist. So, [(24)] cannot be taken to quantify over them. Furthermore, I am unable to express what is asserted in [(24)] with nested slice operators. For I do not remember exactly when I behaved well as a child.' (2007: 77)

As we see it, the case is striking because it calls to mind the case of ignorance of the number of planets, borrowed from Melia, discussed above. There we saw that there are situations of ineliminable ignorance in which we cannot write the requisite paraphrase precisely because we don't know how many planets or how many stars there are. Nonetheless, we have evidence for a theory that specifies that 'the average star has 2.4 planets'. This case is similar because it involves a situation in which we attempt a translation but in which a particular translation (in this case, a translation using slice-operators) is seemingly made impossible because our ignorance prevents the specification of the translation.

But, if the cases are parallel, that is suggestive. Two things ought to be correct. First, there ought to be an analogue of Brogaard's solution available in the number case. Second, the route suggested in the number case ought to be open here. Both are correct. And, at least in part, this explains our preference for the slice-operator route.

What's the analogue of accepting primitive span-operators? So far as we can tell, the analogue would simply be to take the entire number sentence as primitive. Why? Well, we agreed that the way in which to paraphrase 'the average mother has 2.4 children' was to count up all the mothers and all the children. Thus, because we cannot count up either the stars, or the planets, so the sentence as a whole lacks a translation. This suggests that the proper analogue of taking the span-operator as primitive is simply to treat 'the average number of planets orbiting a star is 2.4' as primitive. But this won't do. As good mathematical nominalists, we cannot simply insist that talk that appears to quantify over numbers is just primitively true. At least, it looks a bad option when contrasted with any opposing view.

And, in the mathematical case, we think that we *have* an opposing view that has a good deal to recommend it. The claim we made was that 'the average number of planets orbiting a star is 2.4' is taken to express a sentence that we cannot record in the predicate calculus, because the sentence is too long to write down. We know how to construct it, of course, but its length is prohibitive to it being included in our actual practices. That's not, of course, to treat 'the average number of planets orbiting a star is 2.4' as a primitive term; it's simply to say that, although we know how in principle to write down the required paraphrase, we cannot do so in practice.

So, let's carry the moral across to the temporal case. We agree with Brogaard, as we must, that we don't know precisely when she was well-behaved as a child. Should we then use a primitive span-operator, or a disjunction of many nested slice-operators? Well, as suggested, where we have the option of slices, and not also invoking spans, we prefer to slice—and to restrict our logical machinery to a slice-only approach. It's better because the presentist simply denies that there are such things as spans; they are no part of a presentist ontology (and never have been) and so it will not do to invoke them, alongside another primitive operator.

So, do we have open the option of many, many nested slice-operators? It might be objected that there is a problem of the following sort. Consider the content of what's said. It's easy to say that, at times t1, t2, t3, t4 and t5, I behaved well and, at times t6, t7, and t8, I did not. But that doesn't get us 'usually'. What 'usually' tells us is 'more often than not'. Really, then, simply listing a series of times and saying that 'at those times I behaved well' and then listing another series of times and saying 'at those times I did not' isn't a proper paraphrase of 'usually, I behaved well'.

In lieu of this, consider the following. In addition to the list mentioned, we should add a clause that specifies something like the claim that there were more times at which one is well-behaved than there are times at which one wasn't. We propose the following strategy: rather than talk about times themselves, talk about the paraphrase. More fully: write out the paraphrase mentioned above, that simply generates the list of times. Then say: in the paraphrase, there are more instances of our quantifying over specific slices and attributing good behaviour to Brogaard than there are instances of our quantifying over specific slices and attributing poor behaviour to Brogaard.

Here's a simplified example. Suppose there are five moments that concern us; Brogaard behaved well at three of them and was poorly behaved at the other two. Allow that 'b' names Brogaard and that 'F' stands for the one placed predicate 'behaved well'. Let's further idealise the situation by supposing that we know at which times Brogaard behaved well and at which she did not, in the following way:

(25) *WAS* (∃x (x = b & Fb), &
WAS (∃y (y = b & Fb), &
WAS (∃z (z = b & Fb), &
WAS (∃u (u = b & ~Fb), &
WAS (∃v (v = b & ~Fb))))).

To this we then add: within (25), the paraphrase, that: there are more instances of Fb than ~Fb. This is no more than an instance of the strategy described above. This generates a complete paraphrase.[4] There are no span problems here.[5]

3.2 Continuous Time

[4] We leave out a temporal metric from the tense operator. We suspect that all is required here is that the metric be specified in such a fashion that none of the times are identical. Thus:

(25*) WAS_n (∃x (x = b & Fb), &
$WAS_{m\ (m \neq n)}$ (∃y (y = b & Fb), &
$WAS_{p\ (m \neq p)\ \&\ (n \neq p)}$ (∃z (z = b & Fb), & … *Etc.*

[5] One thought may be that we are over-egging things and that all we require is a paraphrase that says that there are more instances of Fb than ~Fb, if we wish to paraphrase this 'usually' talk. We're sympathetic, but unless we know in principle how to construct the other parts of the paraphrase then it's hard to see how to underpin the truth of 'there are more instances of Fb than ~Fb'. Thus, we include the additional material in the interests of completeness.

Though we think that the above suffices if time is discrete, we may face a challenge if time is continuous. To account for (24), we suggest a strategy according to which we specify that, in the paraphrase, there are more instances of our quantifying over specific slices and attributing good behaviour to Brogaard (Fb) than instances of our quantifying over slices and not attributing good behaviour to them (~Fb). But, if time is continuous, it may seem that there are uncountably many times at which Brogaard is well-behaved and uncountably many times at which she is poorly behaved (assuming that Brogaard ever behaved well and ever behaved poorly). And, if that's correct, then our strategy will fail.

We note two challenging features of this line of objection. First, nothing has been said by presentists as to how they would model continuous time (so far as we know), and so we find ourselves without a body of opinion to fall back on. Second, and relatedly, the way to respond to this challenge will turn on background metaphysical considerations, for the nature of the slices over which we quantify—specifically, their duration and the extrinsic relations in which the slices stand—will feed directly into any response.

By way of illustration, we describe three potential presentist models of continuous time and explain how a version of our approach works. What separates the three models is the duration of an existing background temporal structure: either that structure is maximally extended, minimally extended, or lacks extension. First, consider maximal extension, i.e., a position taken by Zimmerman (2011). Zimmerman endorses the view that only present *objects* exist but there exists a 'permeable cosmic jell-o' (2011: 200) that is identified with space-time. This space-time spans the past, present, and future. If that's the correct background metaphysic for presentism, our task is relatively straightforward. Indeed, we assume that the presentist will say something very similar to the eternalist with this background metaphysic in place. Simply, quantise the space-time and then treat our slice operators as shifting the domain of quantification backwards or forwards by one unit in the resulting quantised space-time.

Of course, if that background metaphysic isn't in place (and we concede that Zimmerman's position is controversial), then matters are more difficult. It isn't at all clear how we can quantise a space-time that doesn't exist, such that we can generate a series of discrete slices over which we may quantify. Nonetheless, here is one way in which a story could be told if we suppose that the present moment is extended, though only to some minimal extent (i.e., a view we associate with Hestevold 2008). This is our second option. In outline, the strategy is to quantise the present moment and treat the resulting moments as what we quantify over in our ordinary discourse about times.

In more detail, assuming that time is continuous, we add that (i) the present is an extended time, (ii) the present is the only time that exists (rejecting Zimmerman's view), and (iii) all objects exist at that time. If time is continuous, then the present (a time), must itself consist in uncountably many times. Now, add the assumption that any x that consists of more than one time *must* have a duration. This yields the result that the present itself has a duration: call it '*D*'. We don't need to take a stand on what *D* is, but a plausible placeholder candidate suggested by current physical theory is a unit measure of Planck time. Call any such moment a 'higher-order' time and specify that no higher-order time contains within it any time that is contained within any other higher-order time. Properly speaking, higher-order times are not the *'times'* of our metaphysical theory. Instead, higher-order times are physically-realised; they emerge from that background metaphysic, and are the 'times' over which ordinary discourse should be taken to quantify

Each use of 'time' in the paraphrase is, thus, a reference to a higher-order time. And so, rather than shifting the domain of quantification form one *time* to another, the span-operators serve to shift the domain of quantification from one higher-order time to the next. Thus, when we speak of Brogaard being well behaved (or not) at times, we should take this to be a reference to these higher-order times. She behaves well at more of these (higher-order times, of which there are countably many) than she behaves poorly. If that is all correct, then our strategy succeeds.

Last, but not least, suppose that the present moment is not extended at all. (As a matter of contingent fact, we aren't aware of a presentist explicitly defending this position.[6]) In this final case, the presentist could respond as follows. Begin by assembling an ersatz time line: a representation of all the times that did exist but don't any more. Array those along some dimension, with the idea being to capture in this representation everything as the B-theoretic eternalist would have it. Then, we may use ordinary measure theory over that representation, to give a measure over pluralities of extensionless slices, some pluralities of which are larger than others (speaking measure-theoretically). If the plurality of slices in which Brogaard was well-behaved is larger than the plurality of slices in which she was not, then we have the desired result.[7]

We concede that this option might be *prima facie* unattractive to the presentist, involving a requirement that to make sense of various parts of everyday discourse we require the notion of an ersatz representation of non-existent times and, further, that the representation has certain features (e.g., that Brogaard is represented as well-behaved at larger regions than she is represented as not well-behaved). But we don't think the presentist need be too troubled by this. It's perfectly reasonable to think that if continuum many times existed then we can represent them as having done so.[8] And, whilst we can see that there's a certain

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6 Some presentists come close. For instance, Trenton Merricks (2007) denies 'that there is anything at all—much less some super-thin slice of being—that is the present time' and says that presentists 'do not believe in a region called the 'present time'' (2007: 125). Merricks is not explicitly committed to a non-extended present, though we think he could accept it, given his background views on presentism.

7 This paragraph is a paraphrase of a suggestion from a referee—for which we are very grateful.

8 We think that this is true independently of whether or not we require truth-makers for truths about what occurs at specific times, and what the nature of those putative truth-makers might be. We don't think that this manoeuvre requires us to be committed to ersatz presentism of the sort defended by Bourne (2006) or Crisp (2007).

dissatisfaction to be had with the fact that this solution solves the problem by appealing to a fact about the size of something that appears in the representation, we exploited something very similar in our original proposal; the fact that our paraphrase included more instances of Fb than ~Fb underpinned the earlier result. It is a feature of our view that the truth of such span claims are understood in terms of features of the paraphrase—itself a form of representation. We don't necessarily see this as a cost, however. That something is not expected from the outset does not make it a bad result. And, as we stressed at the start of the paper, the payoff of our view is that we reduce the number of theoretical primitives.

We close with a concession: we don't know how presentists should model continuous time. We have suggested three options and explained how the presentist can accommodate our solution to the 'span' problem against the background metaphysic. But, very generally, we think that if presentists *can* accommodate continuous time, then they will be able to find a solution to the span problem. After all, even if time is continuous, presentists must find a way to recapture our quantisation of time in our best physical theory, else presentism will be rendered incompatible with our best physical theory. Whichever way the presentist captures that quantisation (as we have tried to in our second option here), we can then put it to work in solving the span problem. And, if it should turn out that time is continuous and that presentism cannot recapture our quantisation of time, then presentism will be shown false—but, plausibly, for reasons other than the problem of span-talk in our everyday discourse.

4. *Concluding Remarks*

We argued that the presentist has a paraphrase of span-talk available to them and that it's a paraphrase strategy that trades solely on the use of slice-operators. We take it that this option is preferable to treating both slice-operators and span-operators as primitive, on the straightforward basis that a system that employs one primitive operator is preferable to a system that employs two (on grounds of theoretical simplicity and elegance).

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